Warm-up!

Let $f(x) =$ $= \frac{(x-\cancel{3})(x+1)}{(x-\cancel{3})} = x+1$

A) Find the zeros of the numerator of

$-1, \quad 3$

B) Find $=2$

C) Find $=3$

D) Find undefined

$f(3) = \frac{\quad}{0}$

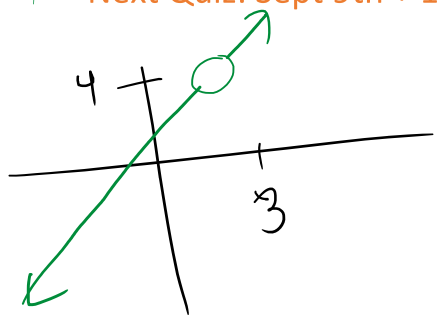
x	y
0	1
1	2
2	3
3	
4	5
5	6

Unit 1 Day 8 (1.9+1.10)

- One Sided Limits
- Vertical Asymptotes
- Holes

Homework: Rational Functions and One Sided Limits

Next Quiz: Sept 9th + 10th



Analyzing Domain

1. Factor Denominator
2. Find Zeros of Denominator
3. Simplify
4. Find Zeros of Denominator

Example:

$$f(x) = \frac{x-3}{x^2-4x+3} = \frac{x-3}{(x-3)(x-1)} = \frac{1}{x-1}$$

$$\frac{1}{3-1} = \frac{1}{2}$$

What is the domain of $f(x)$?

$x \neq 3$, $x \neq 1$

At what x-values is there a vertical asymptote?

$x = 1$

At what x-values is there a hole in the graph?

$\left(3, \frac{1}{2}\right)$

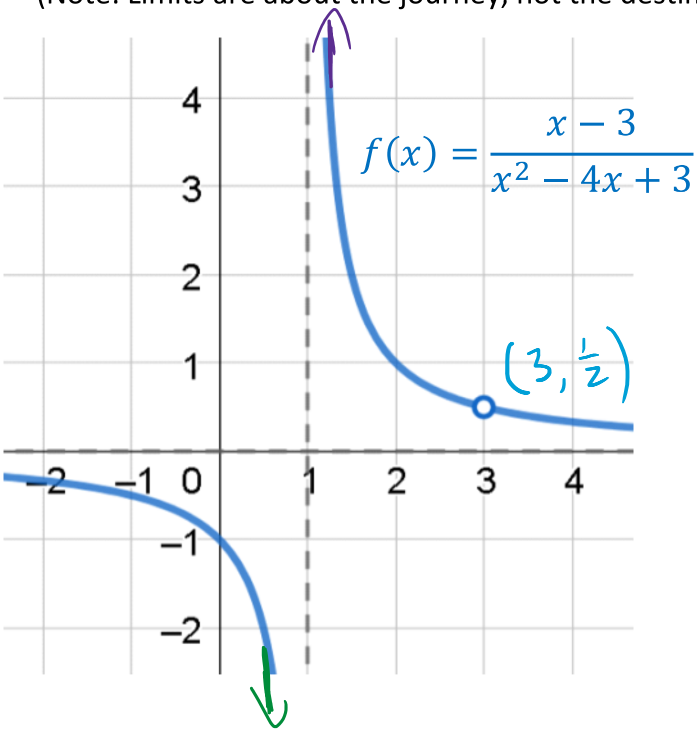
<u>Getting a little more specific</u>

Find the coordinates of the hole in the graph.

Use one-sided limits to describe the behavior of the graph around the vertical asymptote.

One Sided Limits

(Note: Limits are about the journey, not the destination)



$$\lim_{x \to 1^-} f(x) = -\infty$$

x	y

$$\lim_{x \to 1^+} f(x) = \infty$$

x	y

$$f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6} = \frac{(x-2)(x-1)}{(x+3)(x-2)} = \frac{x-1}{x+3} \quad (\text{as long as } x \text{ is not } 2)$$

Find the domain:

$x \neq -3 \quad x \neq 2$

Find the horizontal asymptote (if there is one):

$y = 1$

Find the vertical asymptote(s) (if there are any):

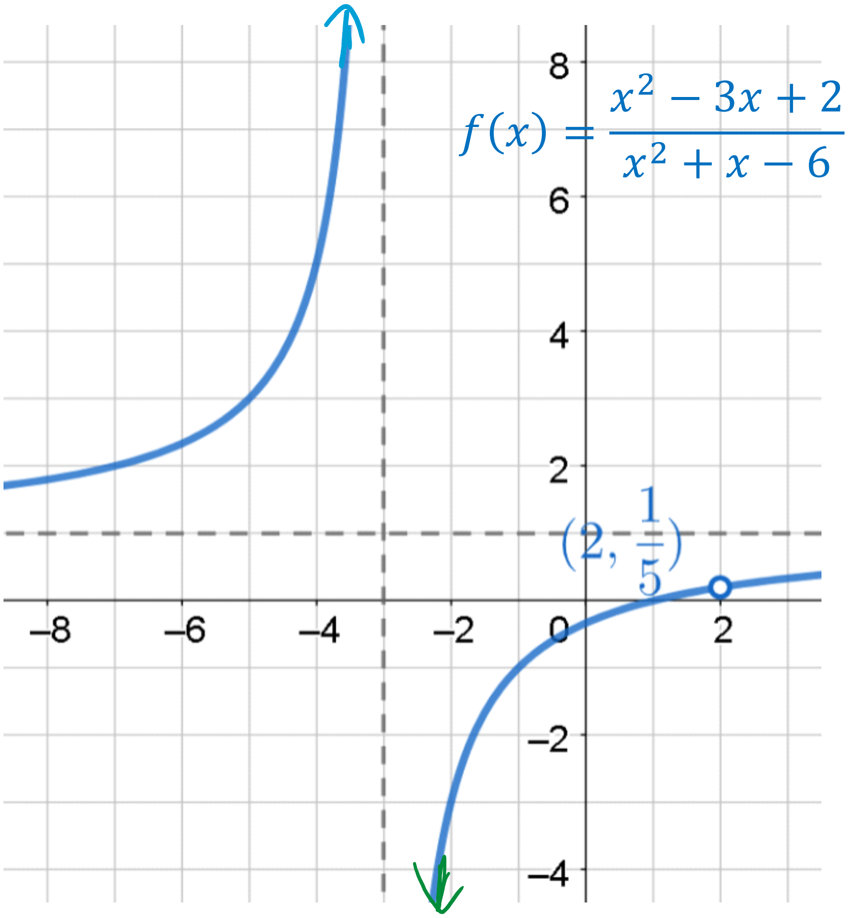
$x = -3$

Find the holes (if there are any):

$\left(2, \frac{1}{5}\right)$

$\frac{2-1}{2+5} =$

I will show you a graph of this, but if you think you can sketch a rough graph, try to beat me to it.

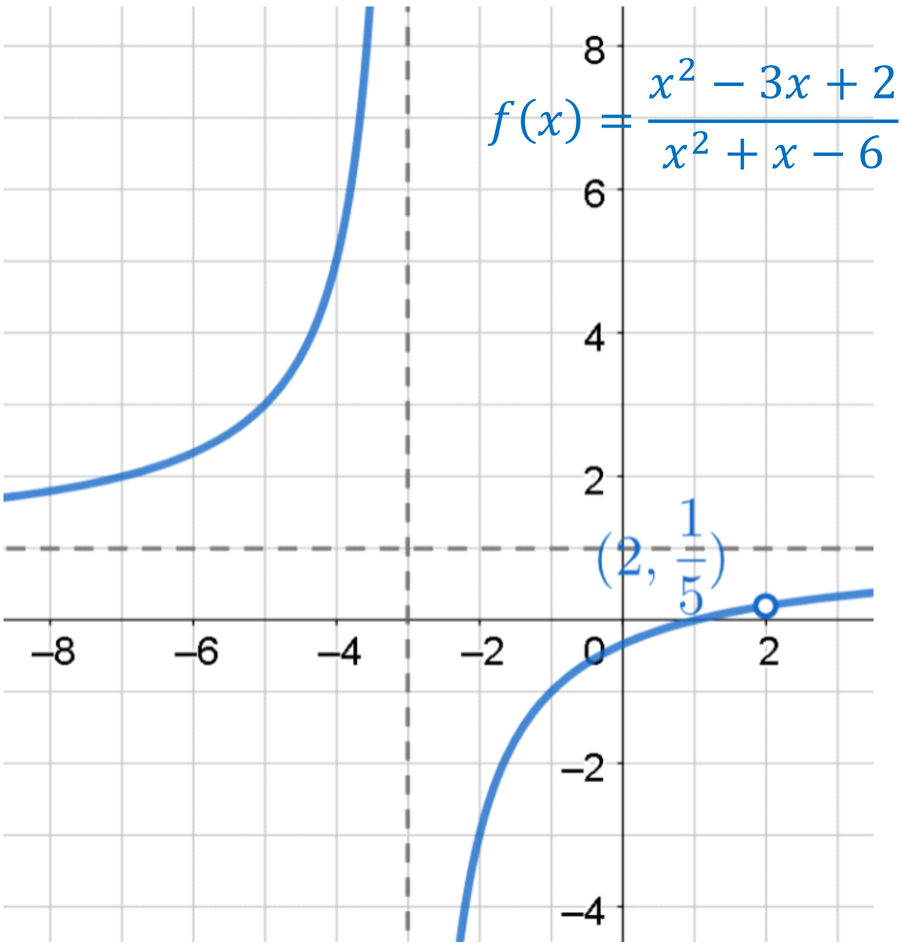


$$\lim_{x \to -3^-} f(x) = \infty$$

$$\lim_{x \to -3^+} f(x) = -\infty$$

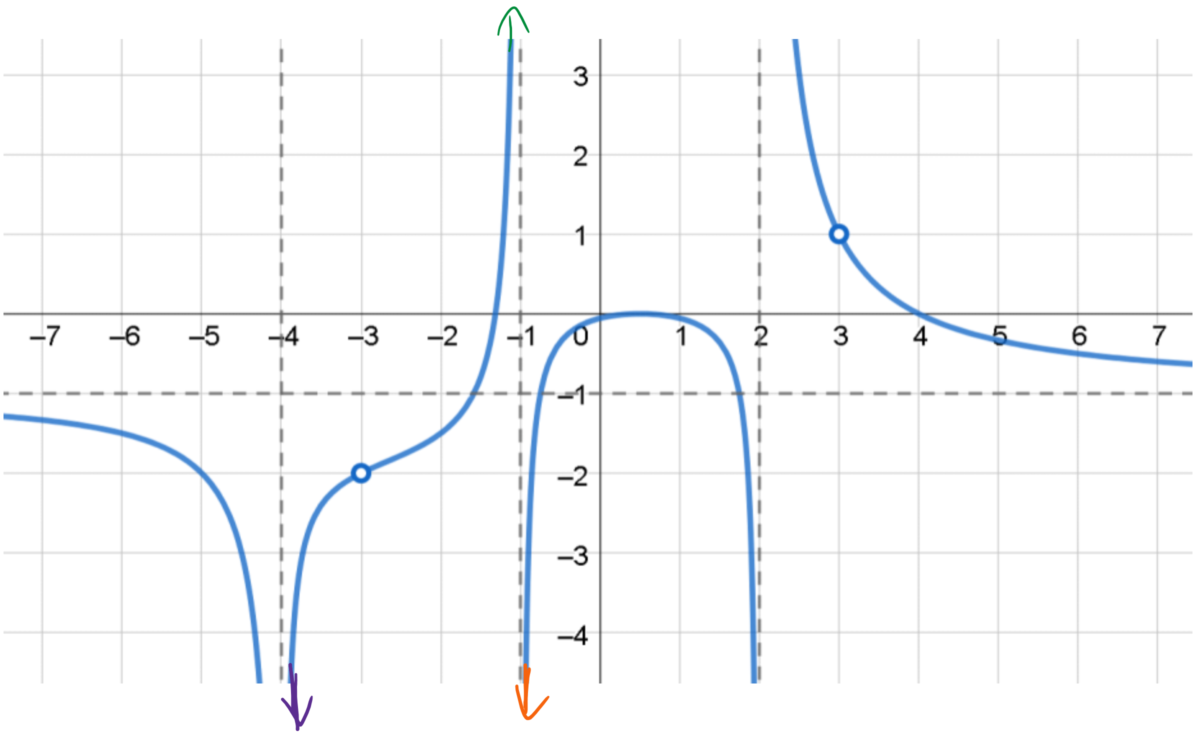
$$\lim_{x \to -\infty} f(x) =$$

$$\lim_{x \to \infty} f(x) =$$



$$\lim_{x \to 2^-} f(x) = \frac{1}{5}$$

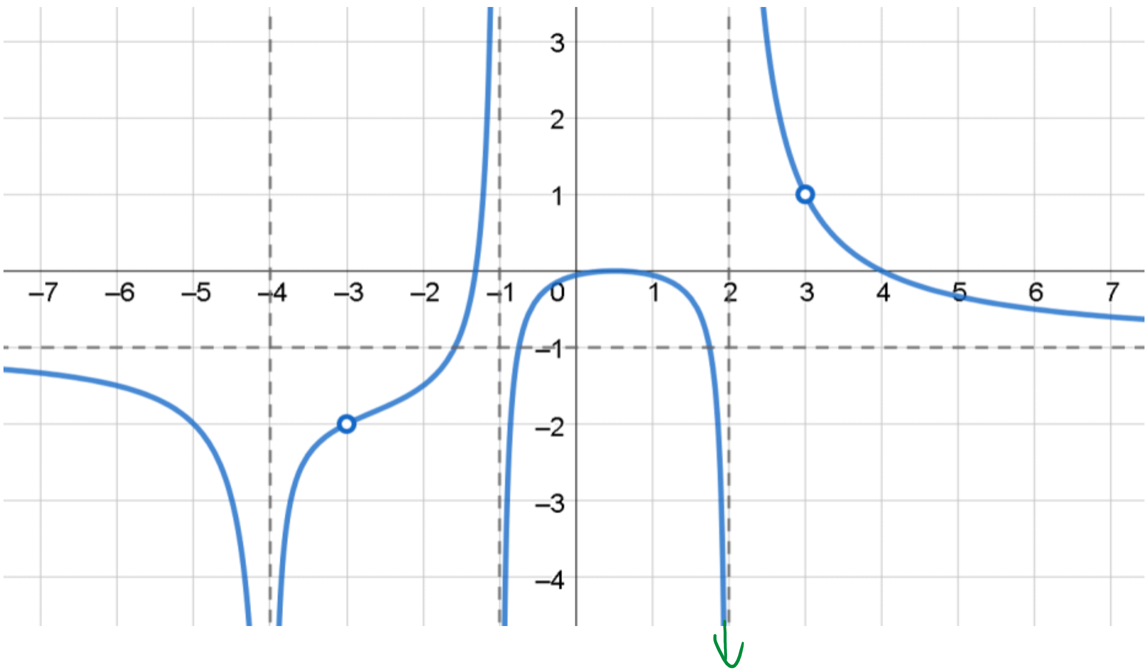
$$\lim_{x \to 2^+} f(x) = \frac{1}{5}$$



$$\lim_{x \to -4^+} f(x) = -\infty$$

$$\lim_{x \to -1^-} f(x) = \infty$$

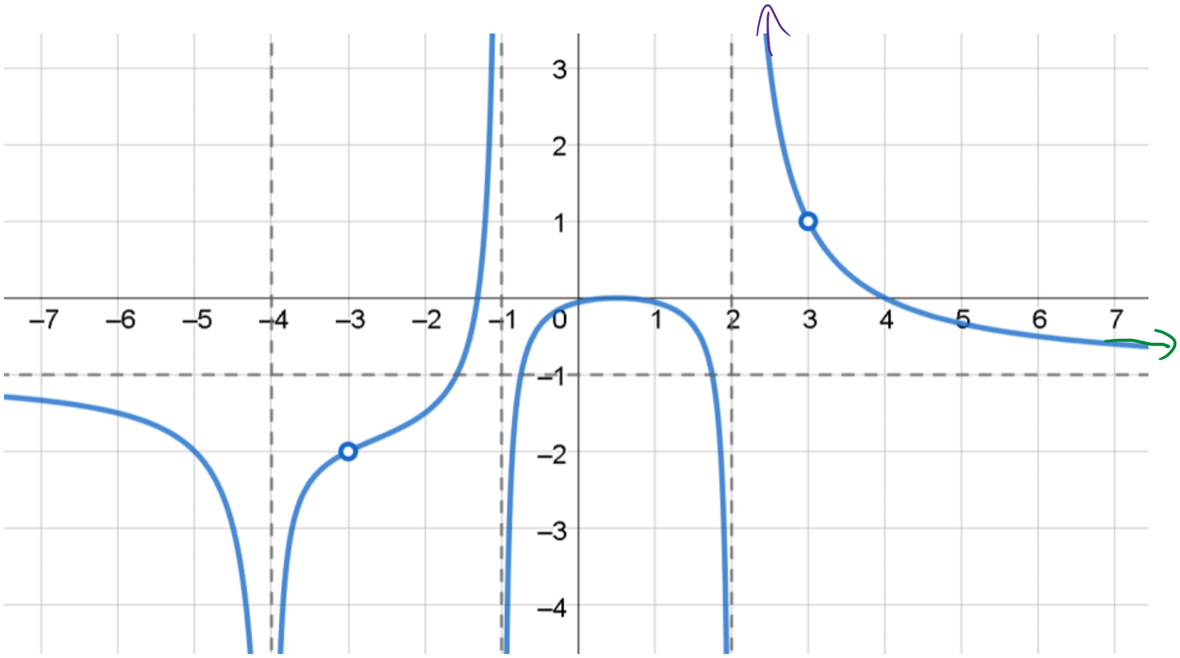
$$\lim_{x \to -1^+} f(x) = -\infty$$



$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = -1$

$\lim_{x \to -3^+} f(x) = -2$



$$\lim_{x\to\infty} f(x) = -1$$

$$\lim_{x\to 3} f(x) = 1$$

$$\lim_{x\to 2^+} f(x) = \infty$$

$$f(x) = \frac{(x^2 - 9)(x + 2)}{(x - 3)(x - 2)(x + 5)} = \frac{(x-3)(x+3)(x+2)}{(x-3)(x-2)(x+5)} = \frac{(x+3)(x+2)}{(x-2)(x+5)}$$

x^3

x^3

Find the domain:

$x \neq 3$, $x \neq 2$, $x \neq -5$

Find the horizontal asymptote (if there is one):

$y = \frac{1}{1} = 1$

Find the vertical asymptote(s) (if there are any):

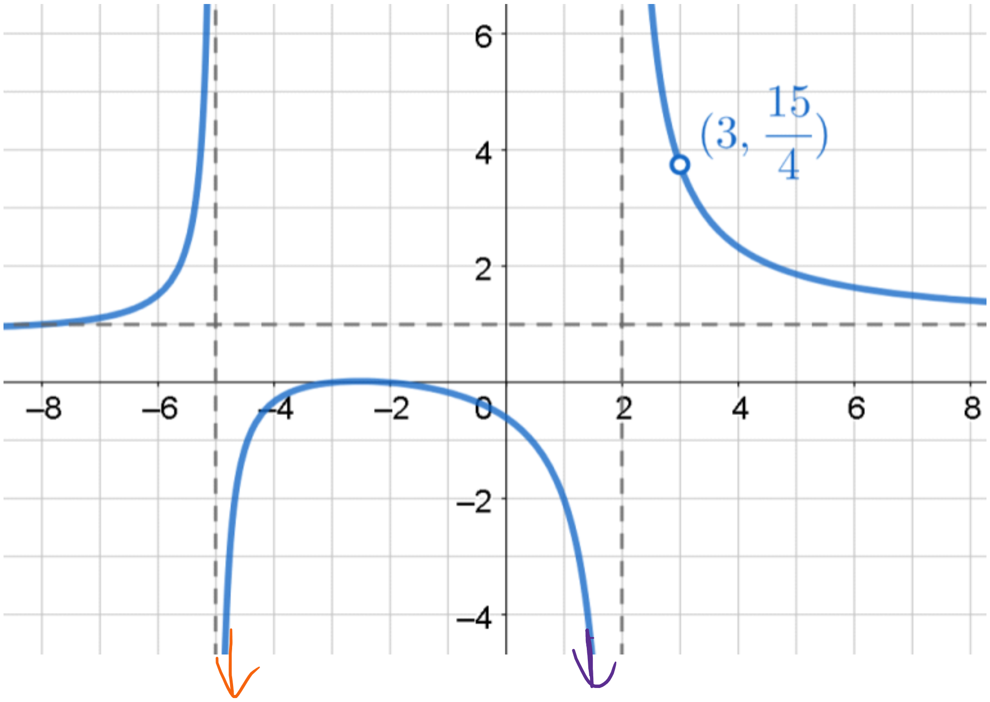
$x = 2$ $x = -5$

Find the holes (if there are any):

$\left(3, \frac{15}{4}\right)$

I will show you a graph of this, but if you think you can sketch a rough graph, try to beat me to it.

$$f(x) = \frac{(x^2 - 9)(x + 2)}{(x - 3)(x - 2)(x + 5)}$$



$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = 1$

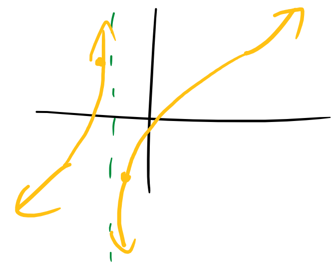
$\lim_{x \to -5^+} f(x) = -\infty$

$\lim_{x \to 3} f(x) = \frac{15}{4}$

$$f(x) = \frac{x^2 + 2x - 3}{x + 2} = \frac{(x+3)(x-1)}{x+2}$$

Find the domain:

$x \neq -2$



$x = -2$

Find the horizontal asymptote (if there is one):

None

Find the vertical asymptote(s) (if there are any):

$x = -2$

Find the holes (if there are any): None

$\frac{x^2}{x} = x$

$\lim_{x \to \infty} f(x) =$

$\lim_{x \to -2^-} f(x) = \infty$

$\lim_{x \to -2^+} f(x) = -\infty$

$\frac{(+)(-)}{(+)}$

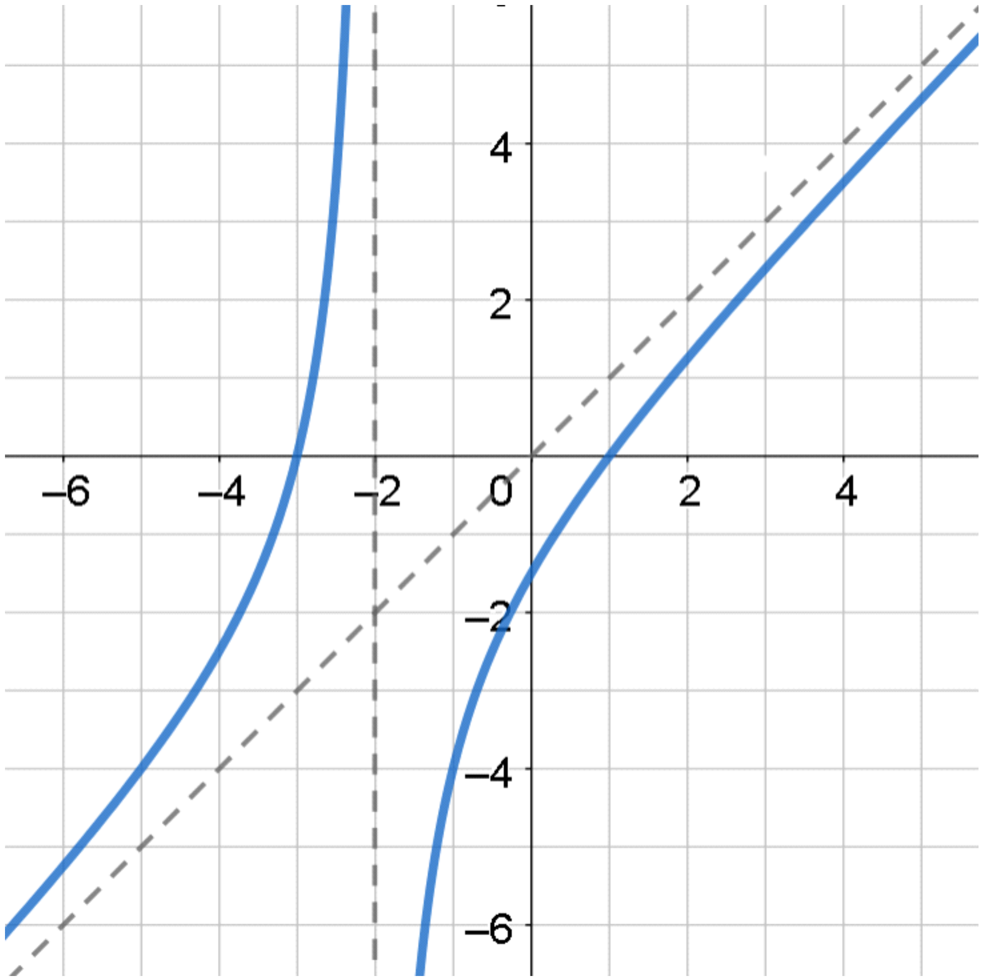
A) $\cancel{3}$

B) ∞

C) $\cancel{-1}$

D) $-\infty$

$$f(x) = \frac{x^2 + 2x - 3}{x + 2}$$



Let's Try It in Our Own Words

If a friend came up to you and asked "When does a rational function have a vertical asymptote at $x = c$?", how would you describe the rule?

VA: $x = -1$

$$\frac{(x+1)(x-2)^3}{(x+1)^2(x-2)^2}$$

$(x+1)(x+1)$

If a friend came up to you and asked "When does a rational function have a hole at $x = c$?", how would you describe the rule?

Let's Try It in Our Own Words

If a friend came up to you and asked "When does a rational function have a vertical asymptote at $x = c$?", how would you describe the rule?

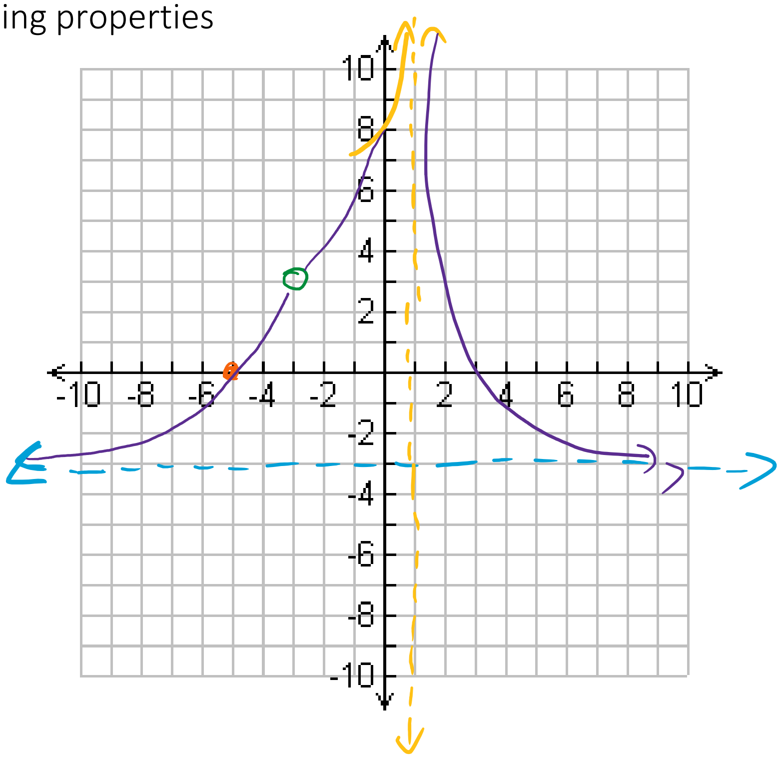
If $x = c$ is a real zero of the denominator but not the numerator, or if $x = c$ is a zero of the denominator and the numerator but the multiplicity is greater as a zero of the denominator.

If a friend came up to you and asked "When does a rational function have a hole at $x = c$?", how would you describe the rule?

If $x = c$ is a zero of the numerator and denominator and the multiplicity of that zero in the numerator is greater than or equal to the multiplicity of that zero in the denominator.

Roughly sketch a graph of $f(x)$ with the following properties

- $f(x)$ is a rational function
- $\lim_{x\to -\infty} f(x) = -3$
- $f(-5) = 0$
- $\lim_{x\to -3} f(x) = 3$
- $\lim_{x\to 1^-} f(x) = \infty$
- $\lim_{x\to 1^+} f(x) = \infty$
- $\lim_{x\to \infty} f(x) = -3$



Roughly sketch a graph of $f(x)$ with the following properties

- $f(x)$ is a rational function
- The numerator of $f(x)$ is cubic
- The denominator of $f(x)$ is quadratic
- $\lim_{x\to\infty} f(x) = -\infty$
- $\lim_{x\to 4^-} f(x) = \infty$
- $\lim_{x\to 4^+} f(x) = -\infty$
- $\lim_{x\to -1^-} f(x) = 2$
- $\lim_{x\to -1^+} f(x) = 2$